THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1520C University Mathematics 2014-2015
Suggested Solution to Test 2

1. (a) $\displaystyle\int \frac{x^2+3x-2}{\sqrt{x}}\,dx = \int x^{3/2}+3x^{1/2}-2x^{-1/2}\,dx = \int \frac{2}{5}x^{5/2}+2x^{3/2}-4x^{1/2}+C$

   (b) $\displaystyle\int e^x + x^e + 2^x\,dx = \int e^x\,dx + \int x^e\,dx + \int 2^x\,dx = e^x + \frac{x^{e+1}}{e+1} + \frac{2^x}{\ln 2} + C$

   (c) $\displaystyle\int 12x(3x^2+1)^{2015}\,dx = \int 2(3x^2+1)^{2015}\,d(3x^2+1) = \frac{1}{1008}(3x^2+1)^{2016}+C$

   (d) Let $u=\sqrt{x}$, then $du = \dfrac{1}{2\sqrt{x}}dx$ and hence $u\,du = \frac{1}{2}\,dx$. Then we have

$$\begin{aligned}\int -\frac{1}{2}e^{\sqrt{x}}\,dx &= \int -ue^u\,du \\ &= \int -u\,d(e^u) \\ &= -ue^u + \int e^u\,du \\ &= -ue^u + e^u + C \\ &= -\sqrt{x}e^{\sqrt{x}} + e^{\sqrt{x}} + C\end{aligned}$$

2. Firstly

$$\begin{aligned}f'(x) &= \int f''(x)\,dx \\ &= \int e^x\,dx \\ &= e^x + A\end{aligned}$$

where A is a constant.

Then

$$\begin{aligned}fx) &= \int f'(x)\,dx \\ &= \int e^x + A\,dx \\ &= e^x + Ax + B\end{aligned}$$

where B is another constant.

Note that, $f(0)=2$ which implies $B=1$ and $f(1)=3+e$ which implies $e+A+B=3+e$ and so $A=2$. Therefore,

$$f(x) = e^x + 2x + 1.$$

3. (a) By long division, $x^3 - 2x + 1 = (x+1)(x^2 - x - 1) + 2$ and so

$$\frac{x^3 - 2x + 1}{x+1} = x^2 - x - 1 + \frac{2}{x+1}.$$

Therefore,

$$\begin{aligned} \int \frac{x^3 - 2x + 1}{x+1}\,dx &= \int \frac{x^3 - 2x + 1}{x+1} = x^2 - x - 1 + \frac{2}{x+1}\,dx \\ &= \frac{x^3}{3} - \frac{x^2}{2} - x + 2\ln|x+1| + C \end{aligned}$$

(b) We want to express $\frac{18-x}{12x^2 - 7x - 12} = \frac{18-x}{(3x-4)(4x+3)}$ into the form $\frac{A}{3x-4} + \frac{B}{4x+3}$, so we have

$$A(4x+3) + B(3x-4) \equiv 18 - x.$$

By comparing coefficient, $A = 2$ and $B = -3$, so

$$\frac{18-x}{12x^2 - 7x - 12} = \frac{2}{3x-4} - \frac{3}{4x+3}.$$

We have

$$\begin{aligned} \int \frac{18-x}{12x^2 - 7x - 12}\,dx &= \int \frac{2}{3x-4} - \frac{3}{4x+3}\,dx \\ &= \frac{2}{3}\ln|3x-4| - \frac{3}{4}\ln|4x+3| + C \end{aligned}$$

4. (a) $\frac{d}{dx}e^{x^2} = 2xe^{x^2}$

(b) By (a), we have $d(\frac{1}{2}e^{x^2}) = xe^{x^2}\,dx$. Therefore,

$$\begin{aligned} \int x^{2n+1}e^{x^2}\,dx &= \int x^{2n}d(\frac{1}{2}e^{x^2}) \\ &= \frac{1}{2}x^{2n}e^{x^2} - \int \frac{1}{2}e^{x^2}\,d(x^{2n}) \\ &= \frac{1}{2}x^{2n}e^{x^2} - \int nx^{2n-1}e^{x^2}\,dx \end{aligned}$$

(c) By using (b) again and again :P , we have

$$\begin{aligned} \int x^9 e^{x^2}\,dx &= \frac{1}{2}x^8e^{x^2} - \int 4x^7e^{x^2}\,dx \\ &= \frac{1}{2}x^8e^{x^2} - 2x^6e^{x^2} + \int 12x^5e^{x^2}\,dx \\ &= \cdots \\ &= \frac{1}{2}x^8e^{x^2} - 2x^6e^{x^2} + 6x^4e^{x^2} - 12x^2e^{x^2} + 24\int xe^{x^2}\,dx \\ &= \frac{1}{2}x^8e^{x^2} - 2x^6e^{x^2} + 6x^4e^{x^2} - 12x^2e^{x^2} + 12e^{x^2} + C \end{aligned}$$